

Heisenberg model of the high-energy hadron collision in terms of chiral fields

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Abstract

Properties of chiral Born-Infeld Theory proposed as the model for shock-wave fireball production in the hadron-hadron collisions was studied. The role of the shock-waves in the multi-particle production was discussed.

1 Introduction and general motivation

In spite the quantum chromodynamics being considerably successful for last 30 years, many opened questions are still waiting for their solution. It is now obvious that the full quantitative theory of strong interactions should be derived from the fundamental QCD Lagrangian and must take into account the most important features of the QCD vacuum, such as the confinement and the chiral symmetry breaking. A number of lattice data, e.g., the disappearance of the chiral condensate and the confinement at the same temperature, is known to point out the deep internal causation between these phenomena.

Another great open problem of the modern particle physics is a saturation of the Froissart bound at high-energy hadron collision. And, as we will see, this problem surprisedly connects very closely to the first one.

Last years (due to the investigation of quark-gluon plasma) a great interest arose to the problem of multi-particle production and the Froissart bound saturation of total cross-sections, first considered by Heisenberg 50 years ago [1].

There are in the order of 3000 charged particles emerging from central collisions at high-energy experiments in RHIC and most of them are pions. It is a very attractive idea to describe this multiple-pion emission in high-energy hadronic and/or nuclear collisions as classical radiation of π -meson field [1]. This idea has been rediscovered and further developed in the modern context of low-energy effective theories [2, 3]. On the other hand, the fireball production problem in Heisenberg picture has been considered in recent papers [4] in the framework of AdS-CFT correspondents. All these results in collection with new experimental data have been stimulating investigation on this subject.

In this work we are going to reformulate Heisenberg's ideas in terms of chiral fields and to find a connection between the concept of the shock-wave fireball production and QCD. The "shock-wave"-like solutions (analogues of the 1+1D solutions in the Heisenberg model) of the Chiral Born-Infeld theory will be studied.

2 Chiral Bag solution of Chiral Born-Infeld Theory

For studying of the process of high-energy hadron collision the idea of fireball production is commonly used. The fireball is a state of the nuclear matter with very high density. Such state usually is simulated by Chiral Bag with quasi-independent quarks and gluons inside. Heisenberg [1] was a first who proposed the model where such object as fireball naturally arises due to shock-wave phenomena.

Heisenberg model is based on Born-Infeld Lagrangian for scalar fields ϕ :

$$\mathcal{L}_{MesonBI} = -\beta^2 \left(1 - \sqrt{1 - \frac{1}{2\beta^2} \phi_{,\mu} \phi^{,\mu}} \right). \quad (1)$$

One should note that this lagrangian can be reformulated as lagrangian of 3D Bosonic String Theory [5] and now this model very widely used in cosmology [6].

The starting point of our consideration is a idea that the fireball at the first moment of the central collision (the state with high density of the nuclear matter) can be treated as the chiral soliton analogously to the nucleon soliton model. Such a state must be very unstable and quickly decay into secondary particles and the Heisenberg model describe this process very well.

Let us consider a direct analogue of the Heisenberg Lagrangian for chiral field

$$\mathcal{L}_{ChBI} = -f_\pi^2 \text{Tr} \beta^2 \left(1 - \sqrt{1 - \frac{1}{2\beta^2} L_\mu L^\mu} \right) \xrightarrow{\beta \rightarrow \infty} -\frac{f_\pi^2}{4} \text{Tr} L_\mu L^\mu, \quad (2)$$

where β is the mass dimensional scale parameter of our model. It can be easily shown that the expansion of the Lagrangian (2) gives us the prototype Weinberg theory as the leading order theory in the parameter β and Heisenberg theory as the leading order theory in the parameter f_π . Moreover this is topologically non-trivial theory and one can identify the topological number of solution with baryon number as usual. Now we consider the spherically symmetrical field configuration

$$U = e^{iF(r)(\vec{n}\vec{\tau})}, \quad \vec{n} = \vec{r}/|r|. \quad (3)$$

Using the variation principle, we get the equation of motion

$$\begin{aligned} & (r^2 - \frac{1}{\beta^2} \sin^2 F) F'' + (2rF' - \sin 2F) - \\ & - \frac{1}{\beta^2} (rF'^3 - F'^2 \sin 2F + 3\frac{1}{r} F' \sin^2 F - \frac{1}{r^2} \sin 2F \sin^2 F) = 0. \end{aligned} \quad (4)$$

Equation (4) has a singular region (singular surface) with singular behavior of solutions. Let r_0 belong to such singular surface. Then

$$\begin{cases} (\beta r_0)^2 - \sin^2 F_0 &= 0 \\ F'_0 \left((F'_0)^2 \sin F_0 \mp F'_0 \sin 2F_0 + \sin^2 F_0 \right) &= 0 \quad \text{where } F_0 = F(r_0). \end{cases} \quad (5)$$

Equations (5) have only two solutions: whether

$$\begin{cases} r_0 &\neq 0, \\ F_0 &= \pm \arcsin(\beta r_0) + \pi N, \quad \text{where } N \in \mathbf{Z}, \\ F'_0 &= 0. \end{cases} \quad (6)$$

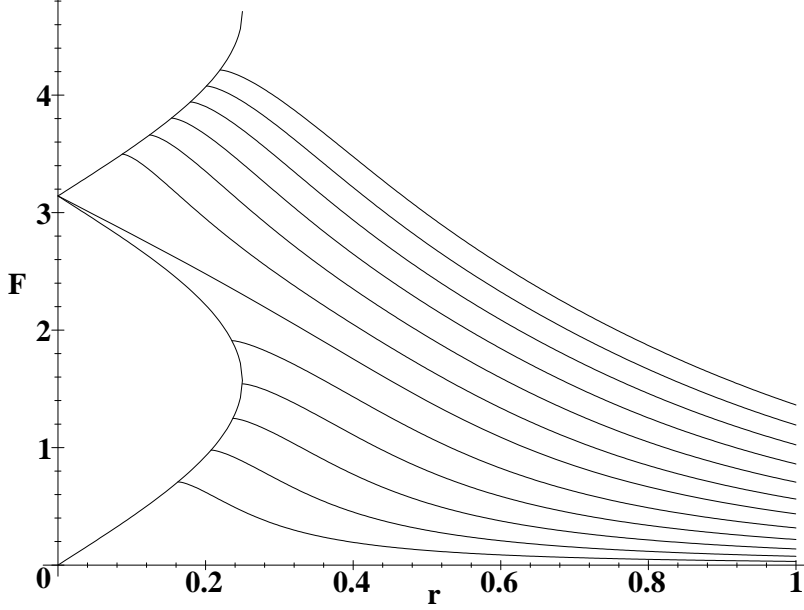


Figure 1: Solutions of equation (9) which have the asymptotics (15) ($a_\infty > 0$) at infinity. Horizontal axis: r (in fm).

or

$$\begin{cases} r_0 = 0, \\ F_0 = \pi N, \text{ where } N \in \mathbf{Z}, \\ F'_0 \neq 0. \end{cases} \quad (7)$$

Topological solitons of ChBI theory correspond to the possibility (7). In my talk I consider the more interesting possibility (6).

One obtains the asymptotic behavior near the singular surface ($r \rightarrow r_0$, $F(r \rightarrow r_0) \rightarrow \arcsin(\beta r_0)$)

$$F(r \rightarrow r_0) = \arcsin(\beta r_0) + \mathbf{b}(r - r_0)^{3/2} + \underline{Q}((r - r_0)^2), \quad (8)$$

where \mathbf{b} is a constant. Of course, the derivative F'_0 at the point $r = r_0$ is zero.

The numerical investigation of the solutions of equation (4) is presented in Fig.1. Most of these solutions can be evaluated only for $r > r_0$, where r_0 is determined by $F(r_0) = \pm \arcsin(\beta r_0)$. But among this set of solutions there are solutions with $r_0 = 0$. Such solutions have the asymptotics

$$F(r) = \pi N + ar - \frac{7a^2 - 4\beta^2}{30(a^2 - \beta^2)}a^3r^3 + \underline{Q}(r^5) \quad (9)$$

at origin ($a^2 < \beta^2/3$ is a constant), and these are the topological solitons of the ChBI theory. The scale parameter $\beta = 807 \text{ MeV}$ is preliminarily defined from the hypothesis that the soliton with $B = 1$ is a nucleon.

Now we would like to draw the attention to another class of solutions. These solutions are defined everywhere, except the small ($\sim 0.2 \text{ fm}$) spherical region about the origin. These solutions

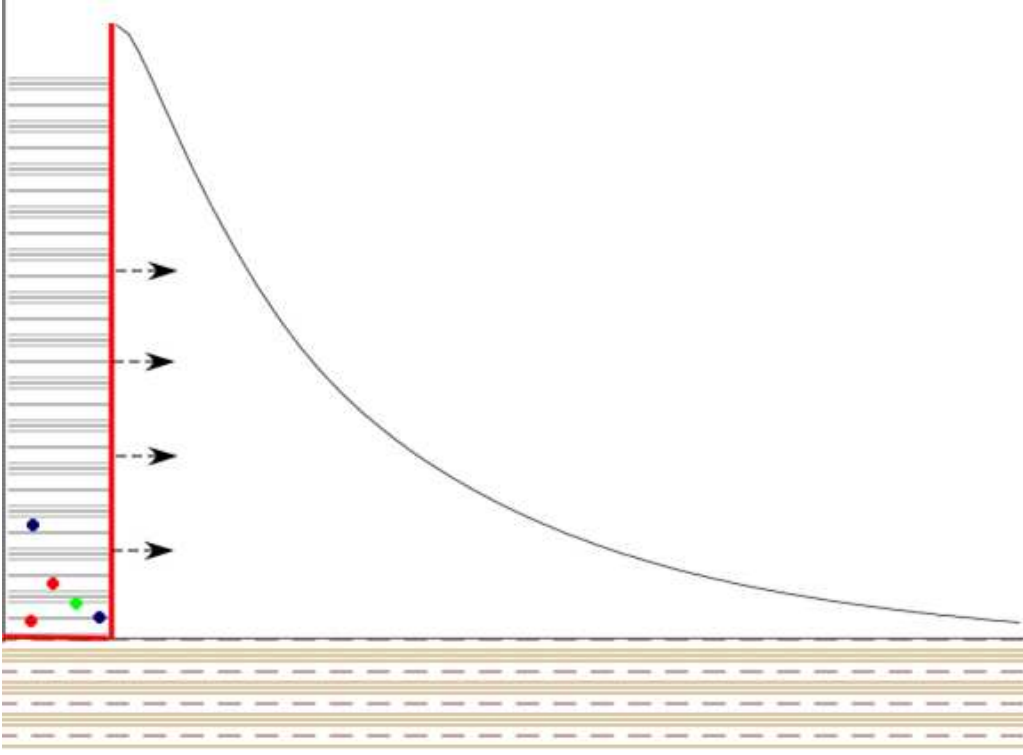


Figure 2: Decay of ChBI fireball.

look like a "bubble" of vacuum in the chiral fields and are of the interest for the chiral bag model. In the internal region ($r < r_0$), the only vacuum configuration can exist. From the mathematical point of view, such "step-like" solutions with jump of $F(r)$ at $r = r_0$ are a generalized solutions.

To clarify the physical nature of this "step-like" generalized solutions, let us consider the projection of the left-hand chiral current on the outward normal of the singular surface. The singular surface of "step-like" solutions is a confinement boundary surface for constituent quarks. For the self-consistency of our "two-phase" picture, let us check is it possible to compensate the non-zero quark chiral current on the confinement surface by the non-zero chiral current of ChBI "step-like" configuration that appear due to the defect on the singular surface. The projection the chiral current on the outward normal of the singular surface for the spherically symmetrical configuration (3) reads

$$(\vec{n} \cdot \vec{J}_\pi)|_{\partial V} = f_\pi^2 \text{Tr} \frac{\tau^a \vec{x} \vec{L}}{\sqrt{1 - \frac{1}{2\beta^2} L_\mu L^\mu}} = \frac{3}{2} f_\pi^2 \frac{\mathbf{b} r_0}{\sqrt{2r_0 + 9r_0^2 \mathbf{b}^2}}, \quad (10)$$

where the coefficient \mathbf{b} from the asymptotic (8) is a function of $F_0 = F(r_0)$ and can be evaluated numerically.

The crucial point for such analysis steams from the fact that $\mathbf{b}(F_0)$ has the singularity at $F_0 = \pi N$ and $r_0 = 0$, or $\mathbf{b}(F(r_0 = 0)) = \infty$. This implies that the soliton solutions of the ChBI theory are solutions with point-like singular chiral source and the "bag"-like solutions are the solutions with the some distribution of the chiral current on the confinement surface. It is possible to show that for any internal constituent quark configuration with confinement inside some volume V the solution of ChBI theory $U(\vec{\pi})$ could be defined which compensate the non-zero

quark's chiral current across the surface ∂V

$$(\vec{n} \vec{J}_q)|_{\partial V} = \sum_g \frac{i}{2} \bar{\Psi}_q(\vec{x} \vec{\gamma}) \gamma_5 \Psi_q = (\vec{n} \vec{J}_\pi)|_{\partial V}. \quad (11)$$

Equation (11) can be considered as a condition on coefficient $\mathbf{b}(\partial V)$, and plays the role of the self-consistency condition between quark and chiral phases.

The qualitative picture of the decay of high density Chiral Born-Infeld soliton (ChiBIon) is follows. As it was shown in [7] the ChiBIon with large topological charge is very unstable ($E_{ChiBIon} \sim B^3$ for the topological charge $B \gg 1$). It is obvious that the most energetically profitable decay is going on via the step-like singularities production (see in Fig. 2). Such singularities should excite the chiral degrees of freedom from vacuum inside the "bubble" and the ChiBIon should explode very quickly.

3 Conclusions

The aim of this paper is to study of "step-like" generalized solutions of the Born-Infeld theory for chiral fields. Physically the critical behavior in the ChBI theory appears when the chiral field strength of the prototype field theory approaches the value of the squared effective coupling constant (β^2).

The Chiral Born-Infeld theory is a good candidate on the role both for the model for the chiral cloud of the baryons and for the fireball. One can show that energy of Chiral Born-Infeld topological solitons grows up B^3 for large topological numbers B . It's means that any compact clusters with a big baryon number are very instable and quickly decay on individual baryons and mesons. The decay of such state is a very stochastic process and leads to generation of large number of hadrons. Details of this process would be the topics for future works.

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